Online Fair Allocations with Binary Valuations and Beyond*

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Abstract

In an online fair allocation problem, a sequence of indivisible items arrives online and needs to be allocated to offline agents immediately and irrevocably. In our paper, we study the online allocation of either goods or chores. We employ popular fairness notions, including envy-freeness up to one item (EF1) and maximin share fairness (MMS) to capture fairness, and utilitarian social welfare (USW) to measure efficiency. For both settings of items, we present a series of positive results regarding the existence of fair and efficient allocations with widely studied classes of additive binary and personalized bi-valued valuation/cost functions. Furthermore, we complement our results by constructing counterexamples to establish our results as among the best guarantees possible.

The fair division of indivisible items is a prominent topic in algorithmic game theory and artificial intelligence, with practical applications. In practice, items are often online, and waiting for all resources to appear can significantly impede progress. We must proactively allocate resources in a timely manner for better outcomes, as in food banks problem, where food arrives online without prior knowledge of future items. Since the food is perishable, it must be allocated upon arrival to some agent. Motivated by these real-world scenarios, we study online fair division: There are n agents, that are known in advance. The items arrive online (the number of items is *unknown*). Upon the arrival of an item, its values to all agents are revealed, and we must allocate it to some agent or discard it immediately and irrevocably. At all times, the allocation among agents must be fair.

Our work is mainly related to the study of Hosseini et al. [2024], which studies the class fairness in online matching for indivisible and divisible items, and present a series of positive results. The model of indivisible items in their paper can be seen as a special case of our model, which is discussed later in this paper. In other words, we study a more general setting, beyond additive binary valuation functions, and derive the approximation results that strengthen their results.

In our paper, we consider MMS (Maximin Share Fairness) along with EF1 (Envy-freeness up to One Good) and the efficiency notion, utilitarian social welfare. We study two settings: goods and chores, in the online fair indivisible items allocation model. In the goods setting, for different classes of valuations, including binary and additive person- alized bi-valued, we introduce the non-wastefulness (NW) notion. We show the inapproximability of EF1 and MMS with general additive valuations, even for only two agents with additive personal tri-valued valuations, and study some restricted valuation functions, such as submodular binary valuation functions. In the chores setting, we introduce the completeness constraints and show the inapproximability of EF1 even for two agents with additive personalized tri-valued cost functions, which blocks our way of studying more general cost functions, and we also investigate the restricted cost functions, such as additive binary cost functions.

*The full paper can be found at https://arxiv.org/abs/2505.24321